Chapter 0 [12 pts]

2. Exercises [12 pts]

0.4 [1] See the student’s solution manual (SSM; www.oxfordtextbooks.co.uk/orc/mqm5e).

0.8 [1] See SSM.

3. Problems [12 pts]

0.1 [2] See SSM.

0.2 [2] When $\beta hc/\lambda \ll 1$ where $\beta = 1/k_B T$ and $k_B$ is the Boltzmann constant (long wavelength limit),

$$e^{\beta hc/\lambda} \equiv 1 + \beta hc / \lambda,$$

thus the Planck distribution is rewritten by,

$$\rho_k = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\beta hc/\lambda} - 1} \equiv \frac{8\pi hc}{\lambda^5} \frac{k_B T \lambda}{hc} = \frac{8\pi}{\lambda^5} k_B T.$$

0.3 [2] According to the Stefan-Boltzmann law, the excitation (M; the power emitted by the area of the emitting region) of a black-body is given by $M = \sigma T^4$ where the Stefan-Boltzman constant ($\sigma$) is $56.7 \text{ [nW/(m}^2\text{K}^4])$ and $T$ is temperature of the black-body. Using the Stefan-Boltzmann law, the power emitted by the sun (the surface area (A) is $6\times10^{18} \text{ [m}^2\text{]}$ and $T = 6000 \text{ [K]}$) ($P_s$) is given by,

$$P_s = M A = 56.7 \times 10^{-9} \text{ [W/(m}^2\text{K}^4])} \times (6\times10^{18}) \text{ [K}^4\text{]} \times 6\times10^{18} \text{ [m}^2\text{] = 4.4\times10^{26} \text{ [W]},}

thus the amount of the energy emitted for 24 hours (86400 [s]) is $4.4\times10^{26} \text{ [W]} \times 86400 \text{ [s] = 3.8\times10^{31} \text{ [J] = 4\times10^{31} \text{ [J].}}$

0.5 [2] Einstein’s heat capacity is given by

$$C_{v,m} = 3 R f_E(T),$$

where $f_E(T) = \left\{ \frac{\theta_E}{T} - \frac{\theta_E}{e^{\theta_E/T}} \right\}^2$.

$\theta_E$ is the Einstein temperature.

(a) In low temperature limit ($T << \theta_E$, $\theta_E/T >> 1$),

$$f_E(T) \approx \left\{ \frac{\theta_E}{T} e^{\theta_E/T} \right\}^2 = \left\{ \frac{\theta_E}{T} e^{-\theta_E/T} \right\}^2 \rightarrow 0$$

Therefore, $C_{v,m} \rightarrow 0$.

(b) In high temperature limit ($T >> \theta_E$, $\theta_E/T << 1$),

$$f_E(T) \approx \left\{ \frac{1}{T} \frac{1}{1 - (1 + \frac{\theta_E}{T})} \right\}^2 = (-1)^2 \rightarrow 1.$$

Therefore, $C_{v,m} \rightarrow 3R$.

0.9 [2] The energy emitted by Na lamp ($P=100 \text{ [W]}$) for 1 [s] ($E_{Na}$) is 100 [J].

The energy of the photon at $\lambda=589 \text{ [nm]}$ ($E_{\text{photon}}$) is $(hc)/\lambda = 3.38\times10^{-19} \text{ [J]}$. Thus, the number of the photon is $E_{Na}/E_{\text{photon}} = 2.96\times10^{20}$. 

Chapter 1 [20 pts]

4. Exercises
1.2 [1] See SSM.
1.3 [1] See SSM.

5. Problems
1.2 [2] \( a(2x-y-z) + b(x+2y-z) + c(-x+y+2z) = 0 \) at \( a = b = c = 1 \), so not linearly independent.
1.3 [3] (a) \( [x,y] = 0 \);
(b) \( [p_x,p_y] = 0 \);
(c) \( [x,p_x] = i\hbar \);
(d) \( [x^n,p_x] = x[x,p_x] + x[p_x,x] = 2i\hbar x \);
(e) \( [x^n,p_x] = x^{n-1}[x,p_x] + x^n [p_x,x] = x^n [x,p_x] x^2 + ... + [x,p_x] x^{n-1} = i\hbar x^{n-1} \)
1.4 [2] See SSM.
1.5 [2] \( [l_y,l_y,l_z] = h^2 l_z \)
1.6 [2] The normalization integral of \( \Psi \) is
\[ \int \Psi^* \Psi \, dx = N^2 \int_{-\infty}^{\infty} e^{-x^2/\Gamma^2} \, dx = N^2 \Gamma \sqrt{\pi} = 1. \]
Therefore, \( N = 1/\sqrt{\Gamma \sqrt{\pi}} \).
When \( x \) is in the range of \(-\Gamma\) and \( \Gamma\), by defining \( x' = x/\Gamma \), the integral is given by,
\[ \int \Psi^* \Psi \, dx = \frac{1}{\Gamma \sqrt{\pi}} \int_{-\Gamma}^{\Gamma} e^{-x'^2/\Gamma^2} \, dx = \frac{1}{\sqrt{\pi}} \int_{-1}^{1} e^{-x'^2} \, dx' = \text{Erf} (1) = 0.84. \]
1.7 [2] See SSM.
1.11 [3] (a) (i) \( [H,p_x] = 0 \), (ii) \( [H,p_x] = [V(x),p_x] = i\hbar k \), (iii) \( [H,p_x] = [V(r),p_x] = \frac{\hbar e^2}{4\pi \epsilon_0} \frac{x}{r^3} \);
(b) (i)(ii) and (iii) \( [H,x] = \frac{\hbar}{im} p_x \)
1.12 [2] \[ \left[ -\frac{\hbar}{i} \frac{\partial}{\partial p_x} , p_x \right] f = i\hbar f. \]

Additional problem [4 pts]
6.a [2] \[ \langle m | \Omega^\Lambda | n \rangle^* = \langle m | \Omega^+ \Lambda | n \rangle = \langle \Omega m \| \Lambda n \rangle^* = \langle \Lambda n \| \Omega m \rangle = \langle n | \Lambda \Omega \| m \rangle. \]
\( \Omega \Lambda \) is not Hermitian.
6.b [2] \[ \langle m | i[\Omega, \Lambda] | n \rangle^* = -i \langle m | \Omega^+ \Lambda - \Lambda^+ \Omega | n \rangle^* = -i (\langle \Omega m \| \Lambda n \rangle^* - \langle \Lambda m \| \Omega n \rangle^* ) \]
\[ = -i \langle n | \Lambda \Omega - \Omega \Lambda \| m \rangle = i \langle n | [\Omega, \Lambda] \| m \rangle. \]
Thus, it is Hermitian.